EXERCISE 2(d)

Use K-map representation to find a minimal sum of the product expression of the following Boolean functions

(1) $F(x,y,z) = \sum_1 m(0,1,5,7) = m_0 + m_1 + m_5 + m_7$

Since it is three variable function. Make K-map of 8 cells as below

$x \backslash yz$	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

$x \backslash yz$	00	01	11	10
0	1	1	0	0
1	0	1	1	0

G_1 G_2

There are two groups G_1 & G_2 of two 1's

In group G_1, x & y remain unchanged (value of both 0)

$\therefore$ G_1 gives the term $x'y'$

In group G_2, x remains unchanged (value 1) & z remains unchanged (value 1) $\therefore$ group G_2 gives the term xz.

$\therefore$ minimal sum of product of $F(x,y,z) = x'y' + xz$ Ans

(ii) $F(x,y,z) = \sum_1 m(1,2,3,6,7) = m_1 + m_2 + m_3 + m_6 + m_7$

K-map for boolean expression is

$x \backslash yz$	00	01	11	10
0	0	1	1	1
1	0	0	1	1

G_1, G_2

In G_1 group, x changes & z changes but y remains unchanged with value 1 $\therefore$ G_1 gives the term y

In group G_2, x remains unchanged (value 0), y changes and z remains unchanged (value 1)

$\therefore$ group G_2 gives the term $x'z$

$\therefore$ $F(x,y,z) = y + x'z$

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$F(x,y,z,w) = \sum m(4,6,7,15) = m_4 + m_6 + m_7 + m_{15}$

no: of variables = 4 ∴ no: of cells in K-map = $2^4 = 16$

K map for 4-variables

xy \ zw	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

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K-map for given Boolean expression is

xy \ zw	00	01	11	10
00	0	0	0	0
01	1	0	1	1
11	0	0	1	0
10	0	0	0	0

G_1 is vertical group, ~~G_2 Horizontal~~ G_2 is left & right square.

G_1 gives the term yzw

G_2 gives the term $x'yw'$

~~G_3 gives the term $x'yw$~~

$\therefore F(x,y,z,w) = x'yz + x'yw' +$

(IV) $F(x,y,z,w) = \sum_1 m(2,3,12,13,14,15)$

K map for the Boolean function is

xy \ zw	00	01	11	10
00	0	0	1	1
01	0	0	0	0
11	1	1	1	1
10	0	0	0	0

G_1

G_2

G_1 gives $x'y'z$

G_2 gives xy

$\therefore F(x,y,z,w) = xy + x'y'z$

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(V) $F(x,y,z) = \sum m(3,5,6,7)$

K-map of given boolean expression is

$x \backslash yz$	00	01	11	10
0	0	0	1 (G_1)	0
1	0	1 (G_2)	1	1 (G_3)

G_1 gives the term yz

G_2 gives the term xz

G_3 gives the term xy

$\therefore \quad F(x,y,z) = xy + yz + xz$

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(VI) $F(x,y,z,w) = \sum m(1, 3, 5, 9, 12, 13, 14)$

K- map for given Boolean expression is

xy \ zw	00	01	11	10
00	0	1	1	0
01	0	1	0	0
11	1	1	0	1
10	0	1	0	0

G_1, G_2, G_3, G_4

In Group G_1, x, y' are unchanged also w is unchanged

$\therefore$ G_1 gives the term $x'y'w$

In group G_2, z and w remains unchanged with values 0 & 1 respectively. Therefore group G_2 given term $z'w$

In group G_3, x, y and z remain unchanged with value 1, 1 & 0 respectively $\therefore$ G_3 gives the term xyz'

In group G_4, which is made of left and right square has x, y & w unchanged with value 1, 1 & 0 respectively $\therefore$ group G_4 gives the term xyw'

So $F(x,y,z,w) = x'y'w + z'w + xyz' + xyw'$

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